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TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 370

STRENGTH TESTS ON PAPER CYLINDERS IN COMPRESSION,  
BENDING, AND SHEAR

By Richard V. Rhode and Eugene E. Lundquist  
Langley Memorial Aeronautical Laboratory

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Washington  
April, 1931

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Summary

Static tests on paper cylinders were conducted at the Langley Memorial Aeronautical Laboratory at Langley Field, Virginia, to obtain qualitative information in connection with a study of the strength of stressed-skin fuselages. The effects of radius-thickness ratio and bulkhead spacing were investigated with the cylinders in compression, bending, combined bending and shear, and torsion.

The results show that:

1. In accordance with the theory of elasticity, unit stresses in pure compression increase with decreasing values of radius-thickness ratio,  $\frac{r}{t}$ , according to the law  $S = \frac{K E}{\frac{r}{t}}$ .

2. Bulkhead spacing has no effect, within the range and accuracy of these tests, upon the strength in compression.

3. Unit stresses in pure bending increase with decreasing values of radius-thickness ratio,  $\frac{r}{t}$ , according to the law  $S = \frac{K E}{\frac{r}{t}}$ .

4. The value of  $K$  in pure bending appears to be about twice the value of  $K$  in pure compression.

5. Bulkhead spacing has no effect, within the range and accuracy of these tests, upon the strength in pure bending.

6. Within the usual range of bulkhead spacing, the effect of bulkhead spacing on shearing strength agrees qualitatively with theory.

7. Bending strength becomes less with the introduction of shear.

8. Shearing strength decreases with the introduction of bending.

### Introduction

In the course of a survey by the National Advisory Committee for Aeronautics of the present status of theoretical and practical knowledge of the strength of stressed-skin structures for airplanes, it has been found that many designers hold irrational opinions as to the behavior of various elements of such structures under load. This confusion of thought is largely a result of the lack of coordinated test data as well as of a lack of understanding of basic theoretical considerations. It was, therefore, considered advisable to make simple qualitative tests on paper cylinders, to check some of the basic theories and to illustrate the behavior of this type of structure under

various conditions of load and with several combinations of some of the principal effective variables. It is planned to extend these tests to metal cylinders at a later date to check the present results and also to obtain usable quantitative data on this type of structure.

Tests were made on some thirty specimens in compression, bending, combined bending and shear, and torsion with several values of bulkhead spacing and radius-thickness ratio for each type of loading.

No attempt was made to perfect a technique for such tests and the results were hurriedly obtained. Sufficient care, however, was taken to insure that a good qualitative picture of the relationships existing would be obtained.

The tests were conducted at the Langley Memorial Aeronautical Laboratory in December, 1930.

#### Apparatus and Method

The models used in these tests were constructed of heavy drawing paper 0.0053 inch thick (average) and 3/8-inch plywood bulkheads cemented with airplane dope. The dimensions of the specimens tested under each type of loading are given in Tables I to IV. Figure 1 shows a group before testing. The procedure followed in building these models was as follows:

The paper was carefully cut into rectangular sheets whose dimensions were such that when rolled into cylinders the proper

diameters and bulkhead spacings would be obtained. One-fourth-inch overlap was allowed for gluing at the longitudinal seam. In all cases the paper rectangles were laid out in such a way that the curvature in the finished cylinders was in the same direction as the curvature in the original paper roll. Also, care was taken that no wrinkles or damages of any kind were suffered by the paper in the course of fabrication.

The test stand, or jig, for loading the cylinders is shown clearly in Figures 14, 19, and 25 with models mounted for various types of loading. These figures are largely self-explanatory and illustrate the methods employed in applying the loads. In the compression tests, the set-up for which is not shown, the cylinders were mounted on top of the platform with their longitudinal axes vertical. Wedges were inserted under the lower bulkhead where necessary to insure uniform bearing. The load was applied to a string which was carefully aligned with the axis of the cylinder and fastened to the center of the upper bulkhead. From this point the string passed down through the cylinder and platform to the weight pan. With this set-up the load remained axial only until the first wrinkle appeared, after which, because of the tilting of the upper bulkhead, it became slightly eccentric.

In applying the load, increments of successively decreasing size were added until failure. This procedure was done as rapidly as possible to avoid errors from variations in the time

the load was applied, although in some cases, for various reasons, short delays occurred during the course of loading.

A flashlight was used to cast a beam of light along the elements of the cylinders. By this means the first wrinkles were readily detected because any deformation caused a definite shadow to be formed.

Relative and absolute humidity measurements were made for each set of tests as a rough check on the relative moisture content of the paper. Variations in humidity were probably sufficiently great from day to day to cause appreciable changes in the moisture content and, hence, variations in the results. Tests with any one type of loading were usually made on the same day, however, to minimize errors from this cause.

In order to determine Young's modulus, a number of long strips of paper were cut both perpendicular and parallel to the axis of the roll and loaded in tension. The elongations of these specimens were measured by means of a transit sighted on a scale attached to the lower end of each strip. It was found that Young's modulus normal to the roll axis was about 3.6 times that parallel to the roll axis, and the average deviation from the mean parallel to the roll axis was 7 per cent. Curves of unit stress versus unit elongation for a number of strips are given in Figure 28.

In computing the unit stress, in every case Young's modulus was taken as 468,000 - which is the average value parallel to

the axis of the roll for the paper used.

### Discussion of Results

The results are given in Tables I to IV and in Figures 29 to 31, inclusive. Photographs of specimens before and after failure are included in Figures 1 to 27.

The collapsing stress for thin-walled circular tubes in either axial compression or pure bending is given by an equation of the form

$$S = \frac{K E}{\frac{r}{t}} \quad (1)$$

where

$S$  = unit compressive stress,

$K$  = a constant,

$E$  = Young's modulus,

$t$  = wall thickness,

$r$  = radius of tube.

Theoretically, the value of  $S$  is independent of bulkhead spacing as long as the cylinder as a whole is not within the Euler range and as long as the elements of the cylinder have no appreciable strength as columns. The cylinders tested in this investigation were constructed within these limitations, so that the results furnish a check on the form of the above equation.

In Figure 29 are plotted the results of the axial compression and pure bending tests. The smooth curves were derived from the formula given above,  $K$  being the average value deter-

mined from the experimental points. It will be noted that the agreement between the experimental points and derived curve is fairly good for both compression and bending. Also, in the case of the compression tests, the effect of bulkhead spacing, if present, is within the experimental error. This is evidenced by the group of points at  $\frac{T}{t} = 955$ , each of which represents a different bulkhead spacing. The influence of bulkhead spacing in the bending tests is likewise within the experimental error.

It is generally believed that the unit stress which can be carried in the extreme fiber in bending is equal to the unit stress which can be carried in axial compression. The results of these tests as shown in Figure 29, throw considerable doubt on this belief, the unit stress in pure bending being about twice as great as the unit stress in direct compression throughout the range of  $\frac{T}{t}$  investigated. This result may be due to the support which the extreme fiber in bending obtains from adjacent fibers which are less severely stressed, or to some other phenomenon.

The results of the pure shear or torsion tests are given in Figure 30. Wagner (Reference 1) states that the collapsing stress for shear is given by an equation of the form

$$S = K_1 E \frac{t}{T} + K_2 E \left(\frac{t}{l}\right)^2 \quad (2)$$

where

$S$  = unit shearing stress,

$l$  = bulkhead spacing,

$K_1$  and  $K_2$  are constants.



The smooth curve given in Figure 30 represents this equation with constants so chosen as to fit the first and third points, which are believed to be the most reliable ones. In Table III it will be noted that the cylinder with the 5-inch bulkhead spacing, representing the second point, is listed as imperfect, which would lead one to believe that the stress carried was somewhat low. The fourth and fifth points, representing the 20-inch and 30-inch bulkhead spacings, are also low. These points are believed to be reliable, but the character of the failure, as seen in Figure 26, is different from that for the shorter cylinders. It is not possible to say, at this time, whether the different type of failure in the long cylinders is a general phenomenon or a result of slightly imperfect specimens or test procedure. It can hardly be said, however, that the results of these tests verify Wagner's curve even qualitatively, except possibly at values of  $\frac{l}{t}$  between 500 and 2000. On the other hand, the results cannot be said to disprove the Wagner formula. Further tests to establish this point will be required and are now contemplated. It is of interest to note that the allowable unit shearing stress begins to rise rapidly in the range of bulkhead spacings now commonly used in monocoque fuselages.

The results of the tests in combined bending and shear are given in Figure 31. As indicated by the upper curve, the presence of shear reduces the strength of the cylinder in bending, probably because a part of the support which the adjacent fibers

give to the extreme fiber is removed when shear is present. Conversely, the presence of a moment reduces the strength in shear, as indicated by the lower curve. It will be noted that the upper curve becomes asymptotic to a unit stress of 325 pounds per square inch, which is the stress carried by the cylinder in pure bending. The lower curve reaches zero at infinity and intersects the y-axis at 162 pounds per square inch, the stress carried by the cylinder in pure shear.

#### C o n c l u s i o n s

It may be concluded from these tests that:

1. In accordance with the theory of elasticity, unit stresses in pure compression increase with decreasing values of  $\frac{r}{t}$  according to the law  $S = \frac{K E}{\frac{r}{t}}$ .

2. Bulkhead spacing has no effect, within the range and accuracy of these tests, upon strength in compression.

3. Unit stresses in pure moment increase with decreasing values of  $\frac{r}{t}$  according to the law  $S = \frac{K E}{\frac{r}{t}}$ .

4. The value of  $K$  in pure bending appears to be about twice the value of  $K$  in pure compression.

5. Bulkhead spacing has no effect, within the range and accuracy of these tests, upon the strength in pure bending.

6. Within the usual range of bulkhead spacing, the effect of bulkhead spacing upon shearing strength agrees qualitatively with theory.

7. Bending strength becomes less with the introduction of shear.

8. Shearing strength decreases with the introduction of bending.

Langley Memorial Aeronautical Laboratory,  
National Advisory Committee for Aeronautics,  
Langley Field, Va., March 10, 1931.

#### REFERENCE

1. Wagner, Herbert : Structures of Thin Sheet Metal, Their Design and Construction. N.A.C.A. Technical Memorandum No. 490, 1928.

TABLE I

Test - Pure Compression

Cylinder	Bulkhead spacing (in.)*	Diameter (in.)	$\frac{r}{t}$	Load at 1st wrinkle (lb.)	Load at failure (lb.)	Unit stress at failure (lb. per sq. in.)	Absolute humidity	Date	Remarks
A	6	6	572	21.4	25.3	256	8.52	12-19-30	O.K.
B	8	8	762	25.1	28.9	219	8.52	12-19-30	O.K.
C	10	10	955	23.8	26.7	162	8.52	12-19-30	O.K.
D	12	12	1144	21.3	21.3	108	8.52	12-19-30	Faulty construction
D	12	12	1144	21.2	24.9	126	7.92	12-22-30	O.K. Rebuilt
E	15	15	1430	18.6	18.6	75	8.52	12-19-30	Faulty construction**
F	2-1/2	10	955		28.8	175	7.92	12-22-30	O.K.
G	5	10	955	29.3	30.2	183	7.92	12-22-30	O.K.
H	7-1/2	10	955	30.2	30.7	186	7.92	12-22-30	O.K.
J	2-1/2	15	1430		24.2	98	7.92	12-22-30	O.K.

\* Measured between the inside faces of the bulkheads.

\*\* It was decided not to rebuild 15-inch cylinders because of difficulties of perfect construction of this large size.

TABLE II

Test - Pure Moment

Cylinder	Bulkhead spacing (in.)	Diameter (in.)	$\frac{r}{t}$	Moment at 1st wrinkle (lb. - in.)	Moment at failure (lb.- in.)	Unit stress at failure (lb. per sq.in.)	Absolute humidity	Date	Remarks
AA	6	6	572	82	87	586	6.15	12-24-30	O.K.
BB	8	8	762	109	117	443	6.15	12-24-30	O.K.
CC	10	10	955	91	143	347	6.15	12-24-30	O.K.
DD	12	12	1144	71	143	241	6.52	12-30-30	O.K.
X	20	10	955	92	130	315	6.15	12-24-30	Model jarred while loaded. Failure possibly premature.
Y	30	10	955	147	154	374	6.15	12-24-30	O.K.

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TABLE III

Test - Torsion\*\*

Cylinder	Bulkhead spacing (in.)	Diameter (in.)	$\frac{l}{t}$	Torque (lb.-in.)	Unit shear- ing stress at failure (lb.per sq.in.)	Computed fail- ing shearing stress (lb. per sq. in.)*	Absolute humidity	Date	Remarks
Ta	2-1/2	10	477	210	259	259.0	6.8	1-7-31	O.K. 18 wrinkles uniformly spaced.
Tb	5	10	955	123	152	181.7	6.8	1-7-31	Cylinder imperfect 13 wrinkles.
Tc	10	10	1910	131	162	162.0	6.8	1-7-31	O.K. 11 wrinkles.
U	20	10	3820	117	145	157.6	6.8	1-7-31	O.K.
V	30	10	5730	84	104	156.7	6.8	1-7-31	O.K.

$$* S = .318 E \frac{t}{r} + 50 E \left(\frac{t}{l}\right)^2$$

$$** \frac{r}{t} \text{ constant at } 955.$$

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TABLE IV\*

Test - Moment and Shear

Cylinder	Bulkhead spacing (in.)	Diameter (in.)	Distance from load to support (in.)	Distance from point of failure to sup- port (in.)	Distance from load to point of failure (in.)	Shear at failing load (lb.)	Moment at point of failure (lb.-in.)	$\frac{\text{moment}}{\text{radius} \times \text{shear}}$	Maximum unit shear- ing stress at fail- ure (lb. per sq.in.)	Maximum unit compres- sive stress at fail- ure (lb. per sq. in.)	Absolute humidity	Date	Remarks
M	10	10	10-3/4	2.01	8.74	7.91	68.1	1.72	96.0	165	6.52	12-30-30	O.K.
N	10	10	20-3/4	3.16	17.59	4.43	70.9	3.35	53.8	172	6.52	12-30-30	O.K.
O	10	10	30-3/4	1.94	28.81	5.28	138.2	5.54	64.0	336	6.52	12-30-30	O.K.
P	10	10	40-3/4	3.87	36.88	3.05	114.5	7.50	37.0	278	5.29	12-31-30	O.K.
Q	10	10	10-3/4	1.58	9.17	7.23	65.9	1.82	87.7	160	5.29	12-31-30	O.K.

\*  $\frac{r}{t}$  constant at 955.



Fig.1 Models A,B,C,D, and E before test.

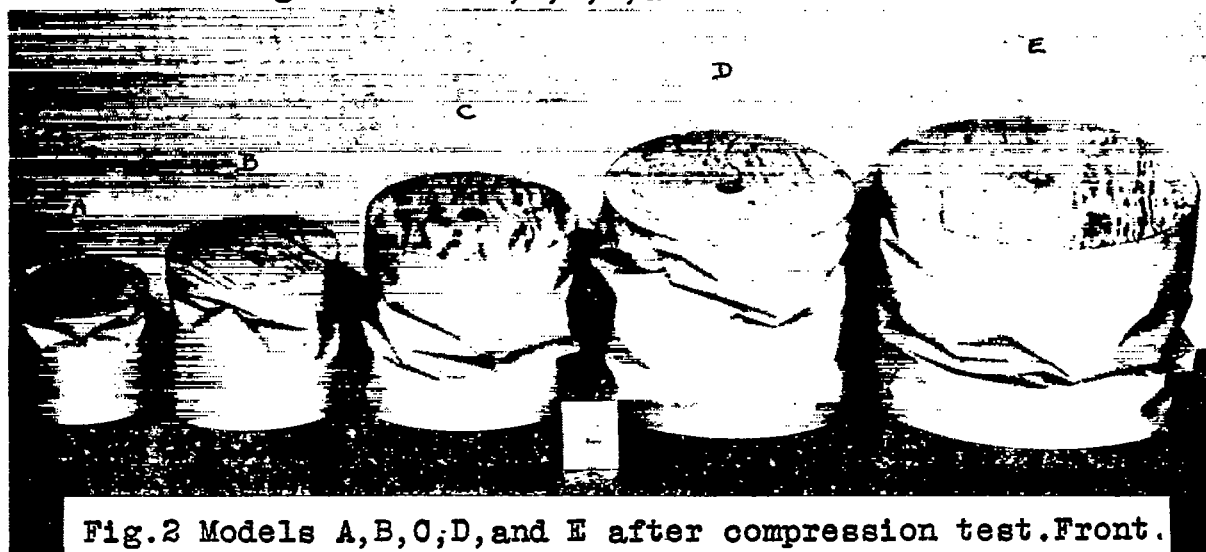


Fig.2 Models A,B,C,D, and E after compression test. Front.

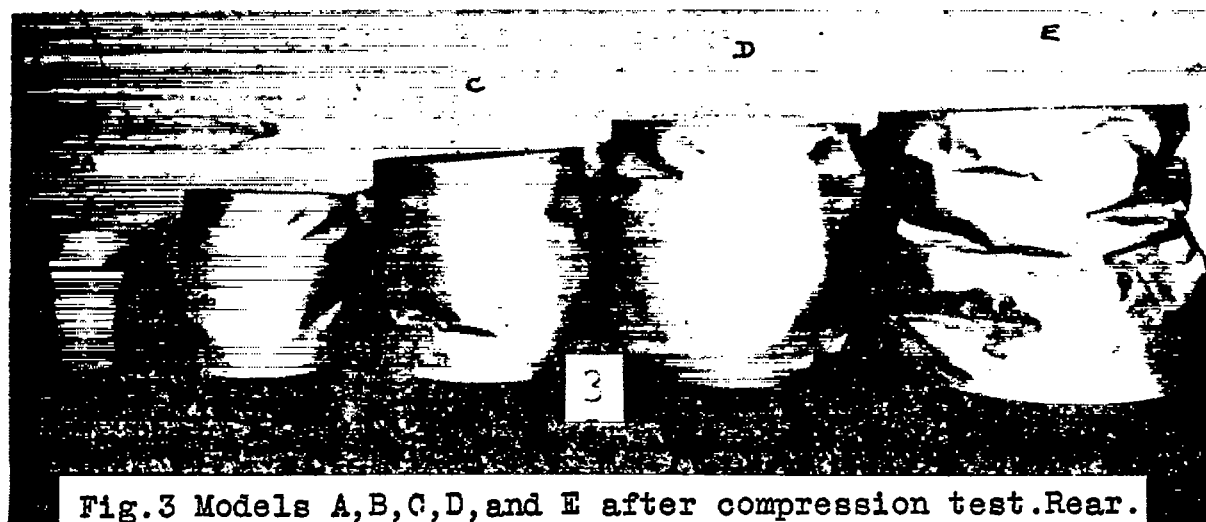


Fig.3 Models A,B,C,D, and E after compression test. Rear.



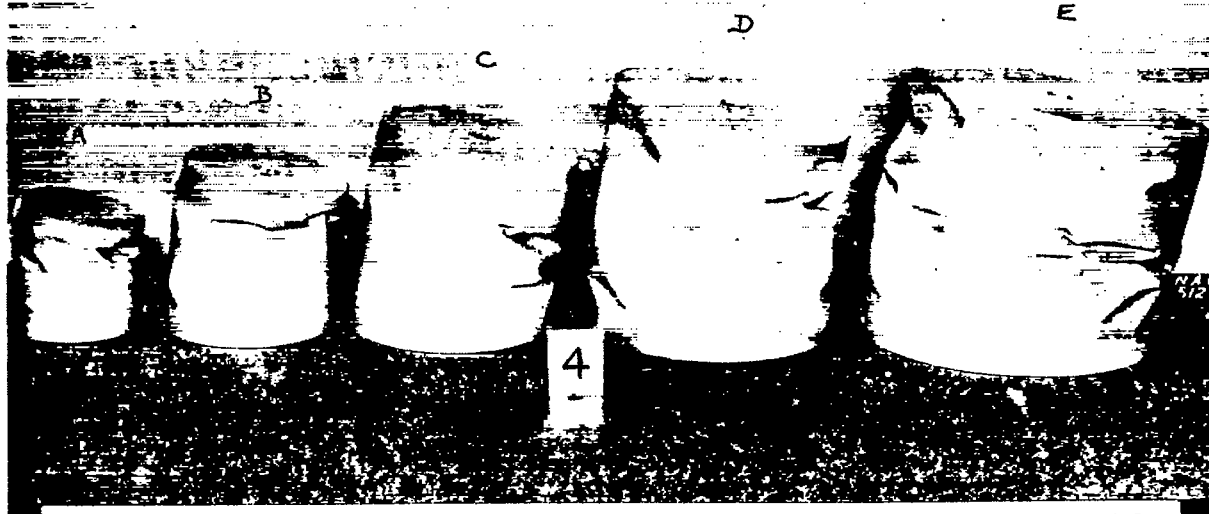


Fig.4 Models A,B,C,D, and E after compression test.Left side.

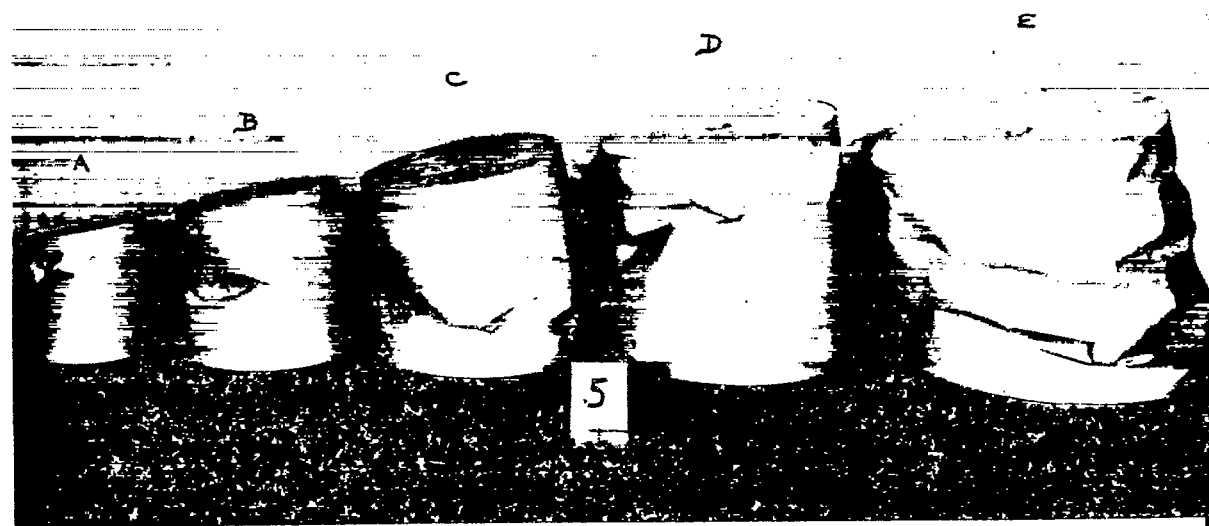


Fig.5 Models A,B,C,D, and E after compression test.Right side.

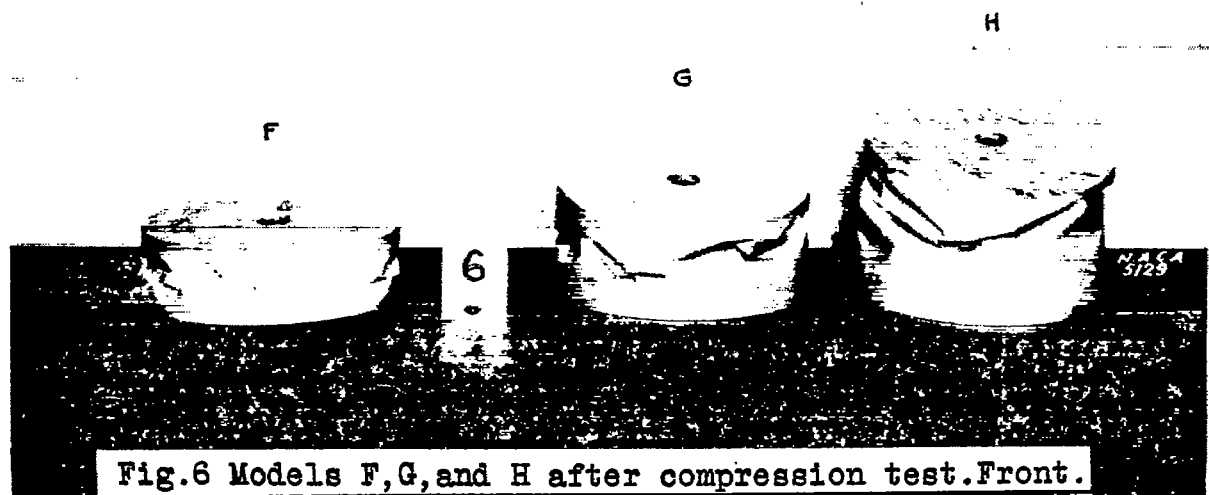


Fig.6 Models F,G, and H after compression test.Front.

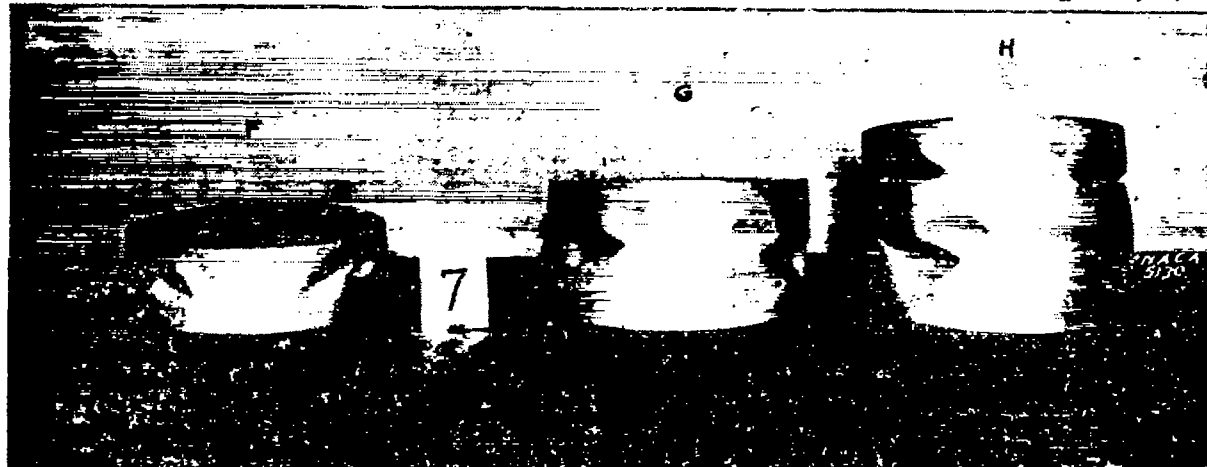


Fig.7 Models F,G,and H after compression test.Rear.



Fig.8 Models F,G,and H after compression test.Left side.

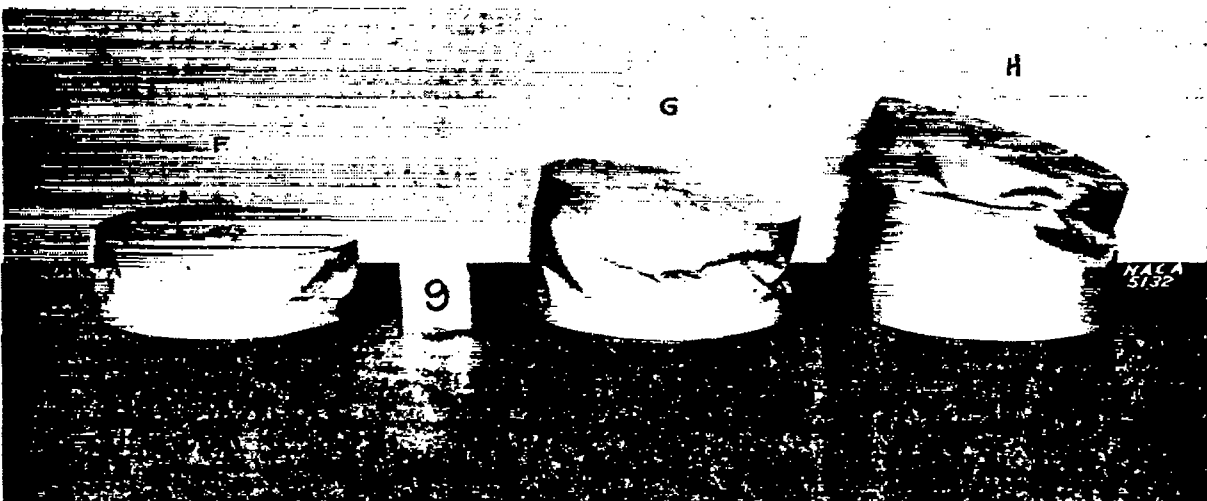


Fig.9 Models F,G,and H after compression test.Right side.

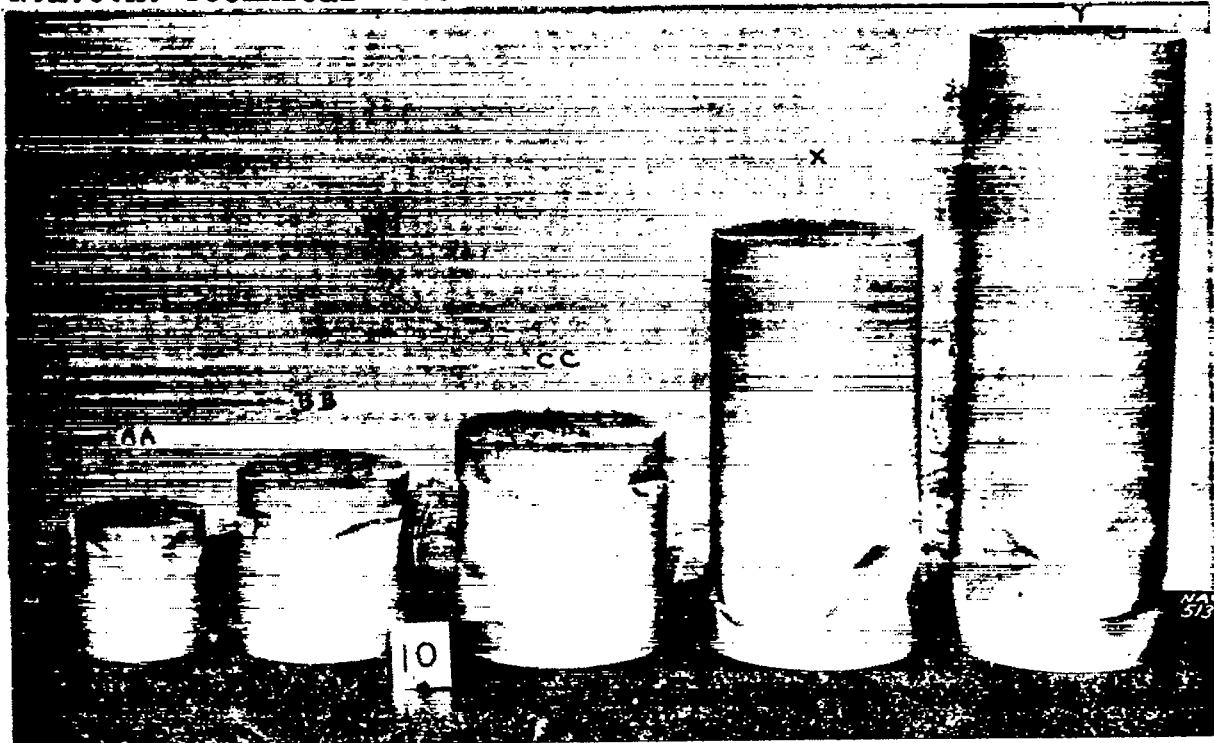


Fig.10 Models AA,BB,CC,X,and Y after pure bending test.Front.

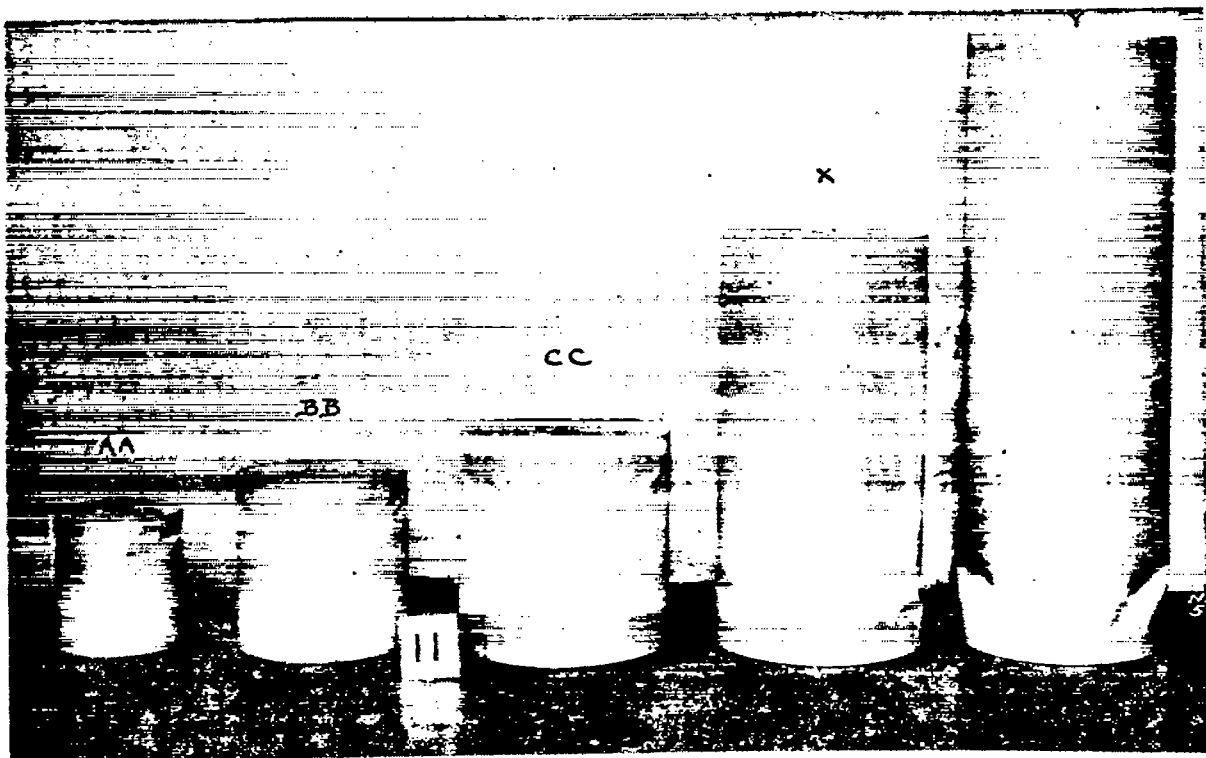


Fig.11 Models AA,BB,CC,X,and Y after pure bending test.Rear.

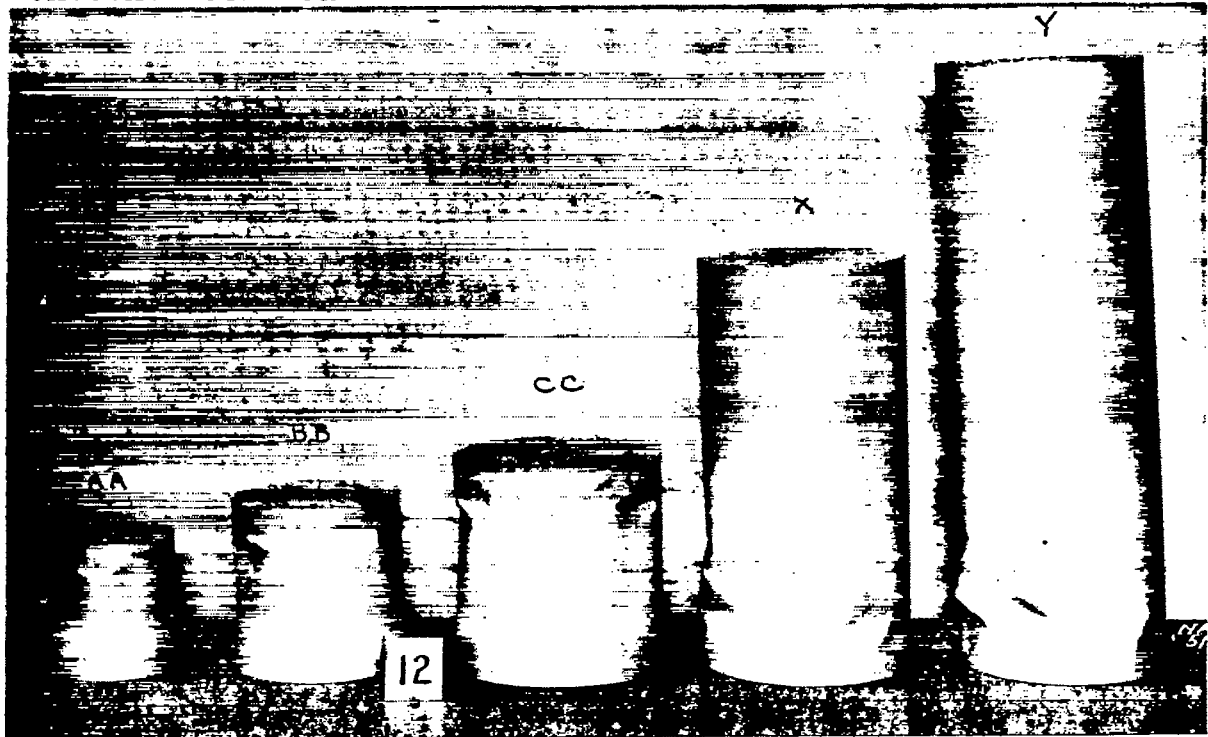


Fig.12 Models AA,BB,CC,X and Y after pure bending test.  
Left side.

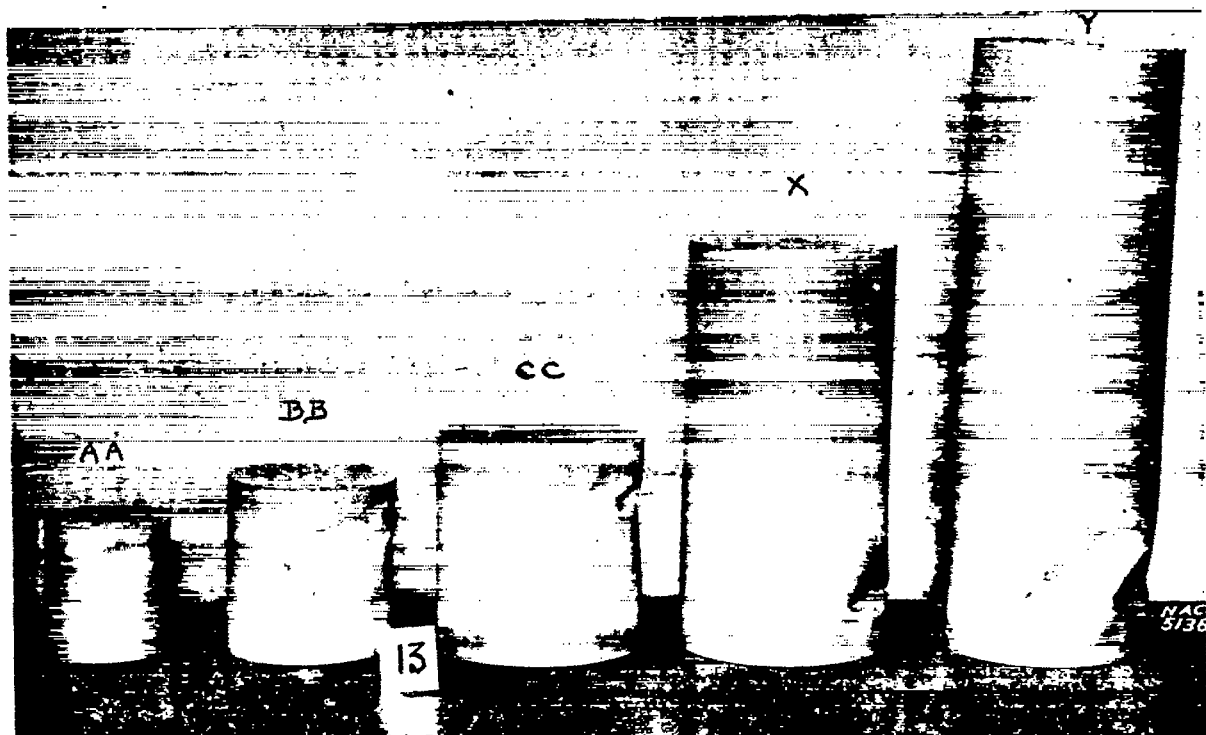


Fig.13 Models AA,BB,CC,X and Y after pure bending test.  
Right side.

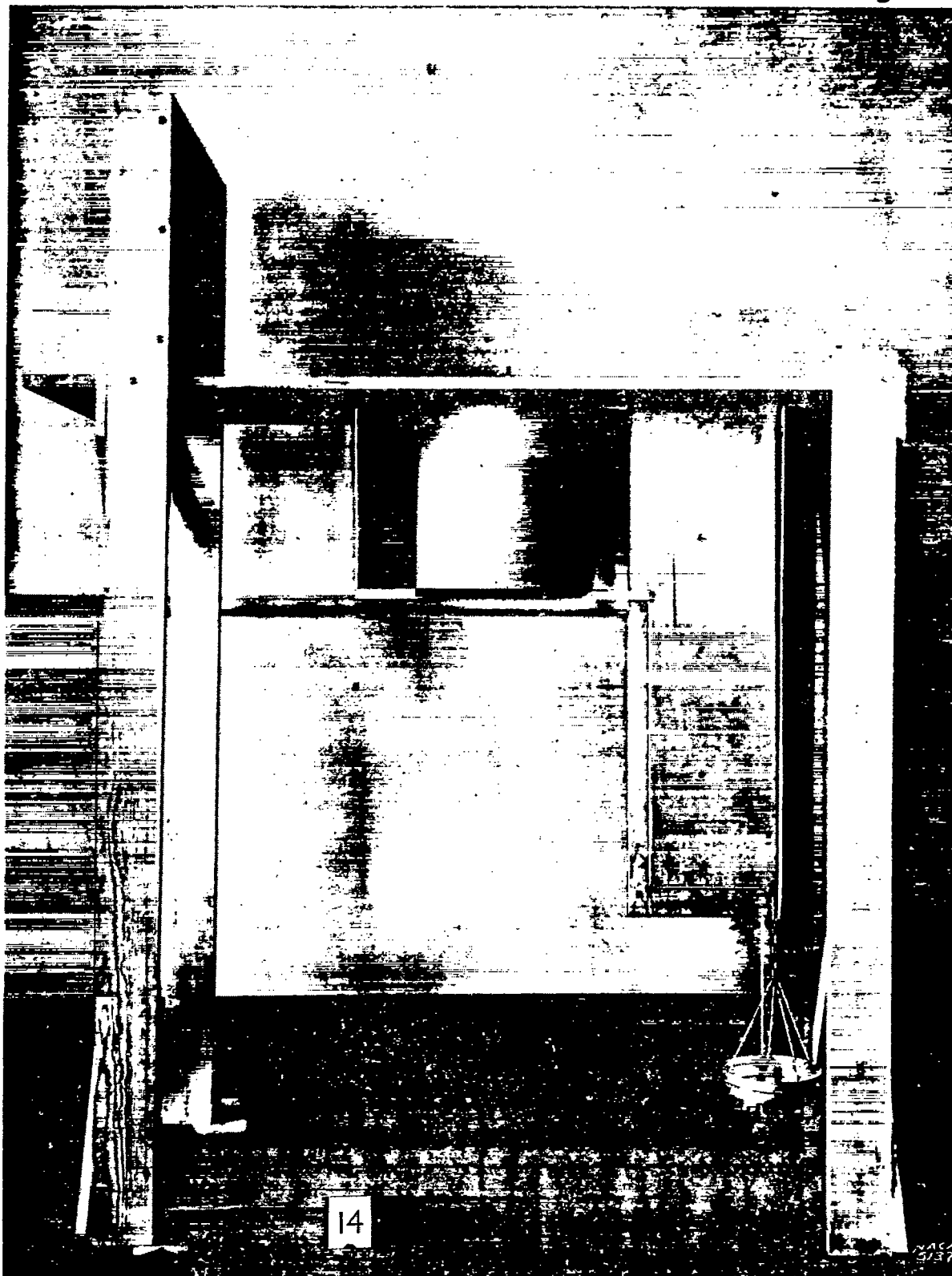


Fig.14 Testing apparatus with model in place.(Pure bending).

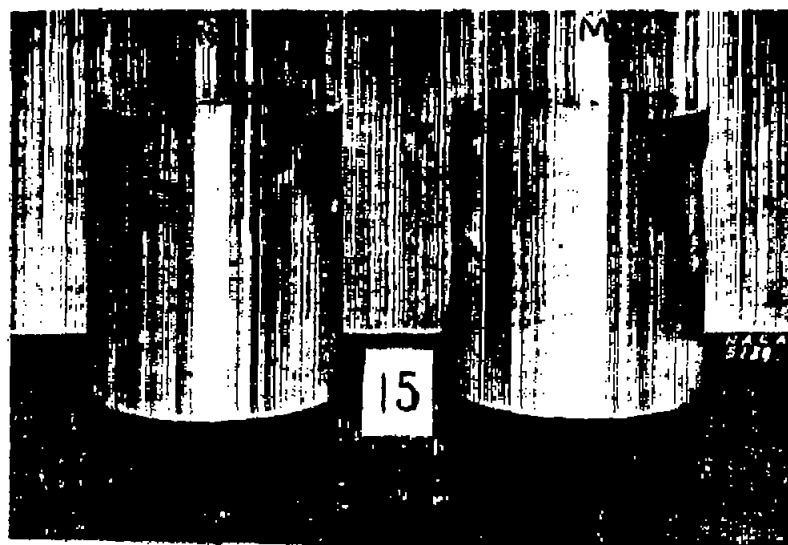


Fig.15 Models M and N after test in combined pure bending and shear.Front.



Fig.17 Models M and N after test in combined pure bending and shear.Left side.



Fig.16 Models M and N after test in combined pure bending and shear.Rear.



Fig.18 Models M and N after test in combined pure bending and shear,Right side.

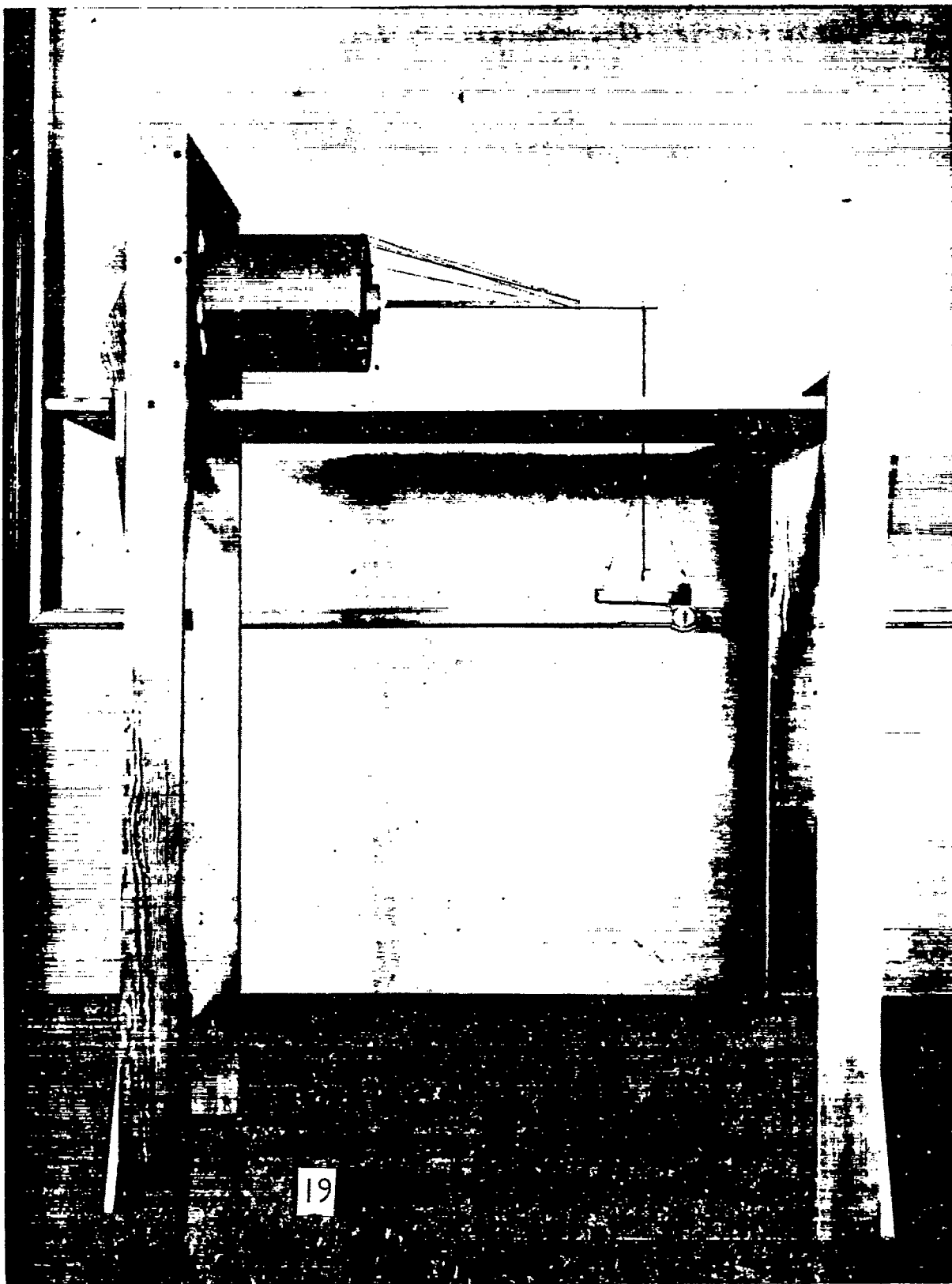


Fig.19 Apparatus with model in place.(Pure bending and shear.)



Fig.20 Models O,P,and Q after test in combined pure bending and shear.Front.

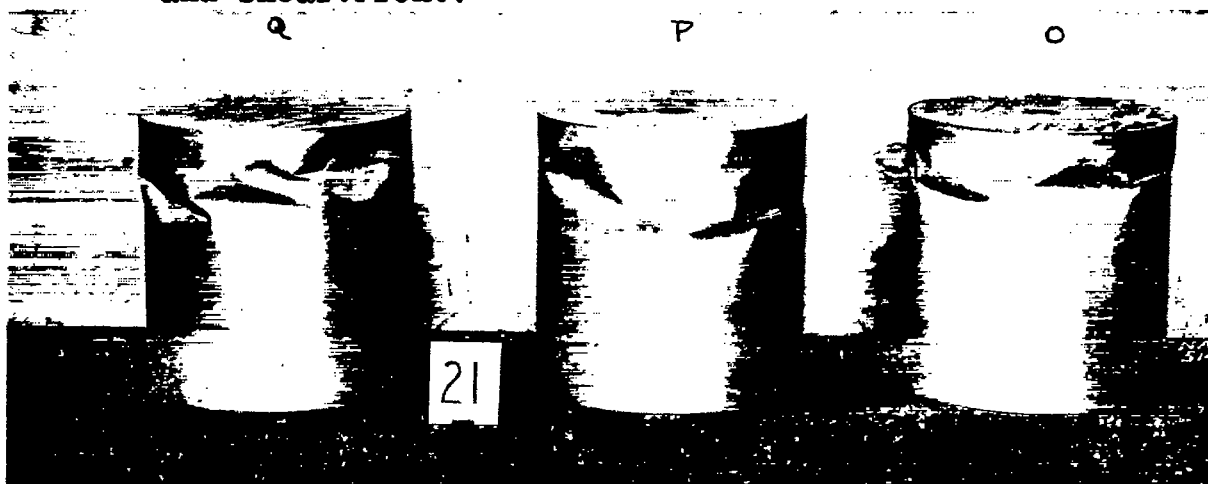


Fig.21 Models O,P,and Q after test in combined pure bending and shear.Rear.

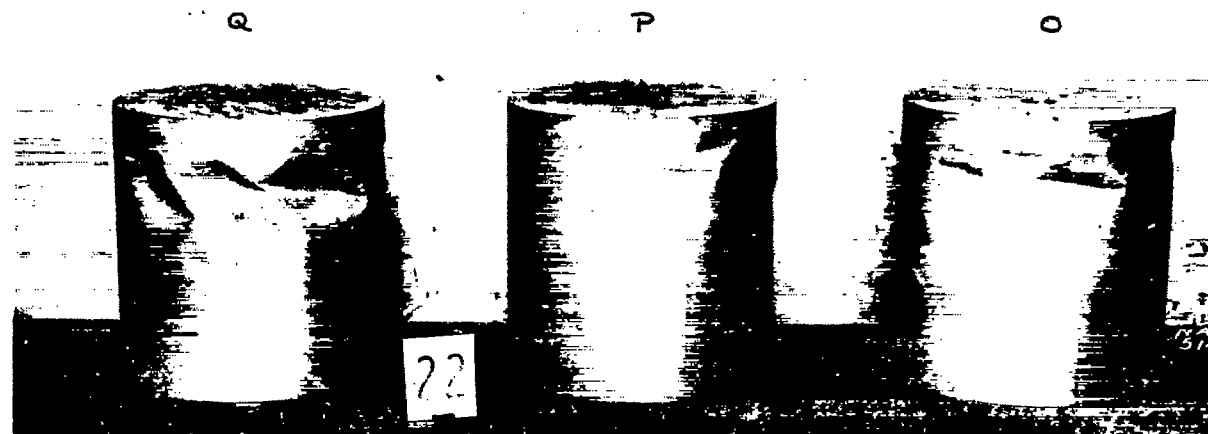


Fig.22 Models O,P,and Q after test in combined pure bending and shear.Left side.





Fig.23 Models O,P,and Q after test in combined pure bending and shear.Right side.

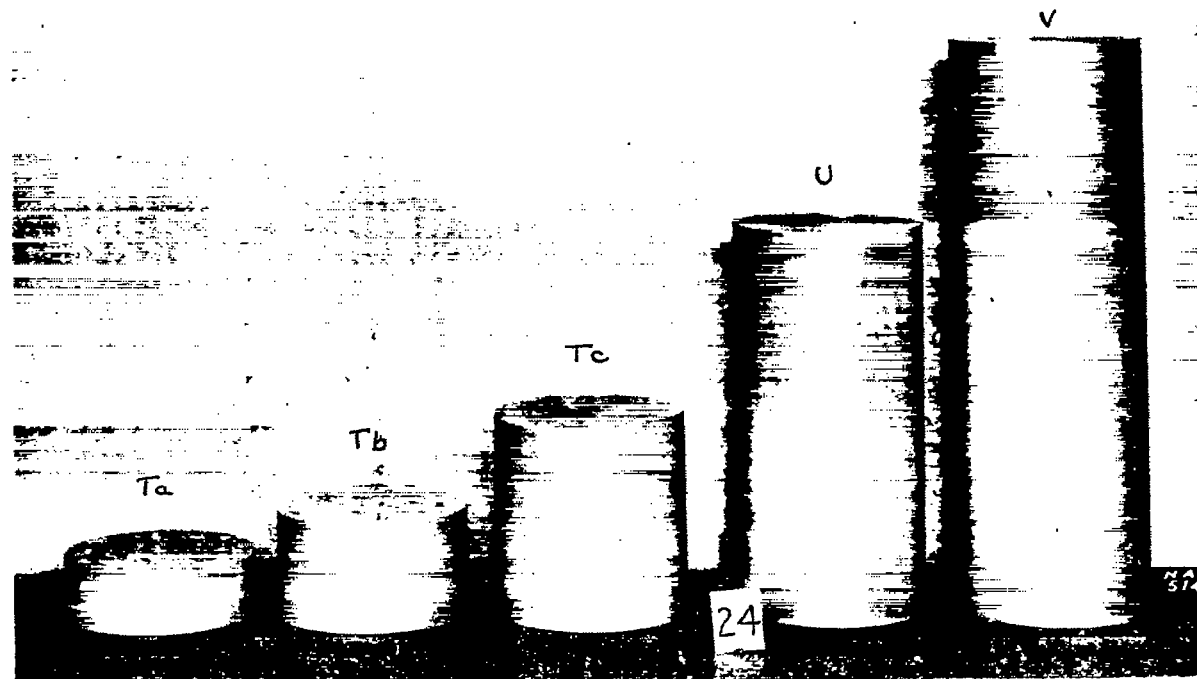


Fig.24 Models Ta,Tb,Tc,U,and V before testing.

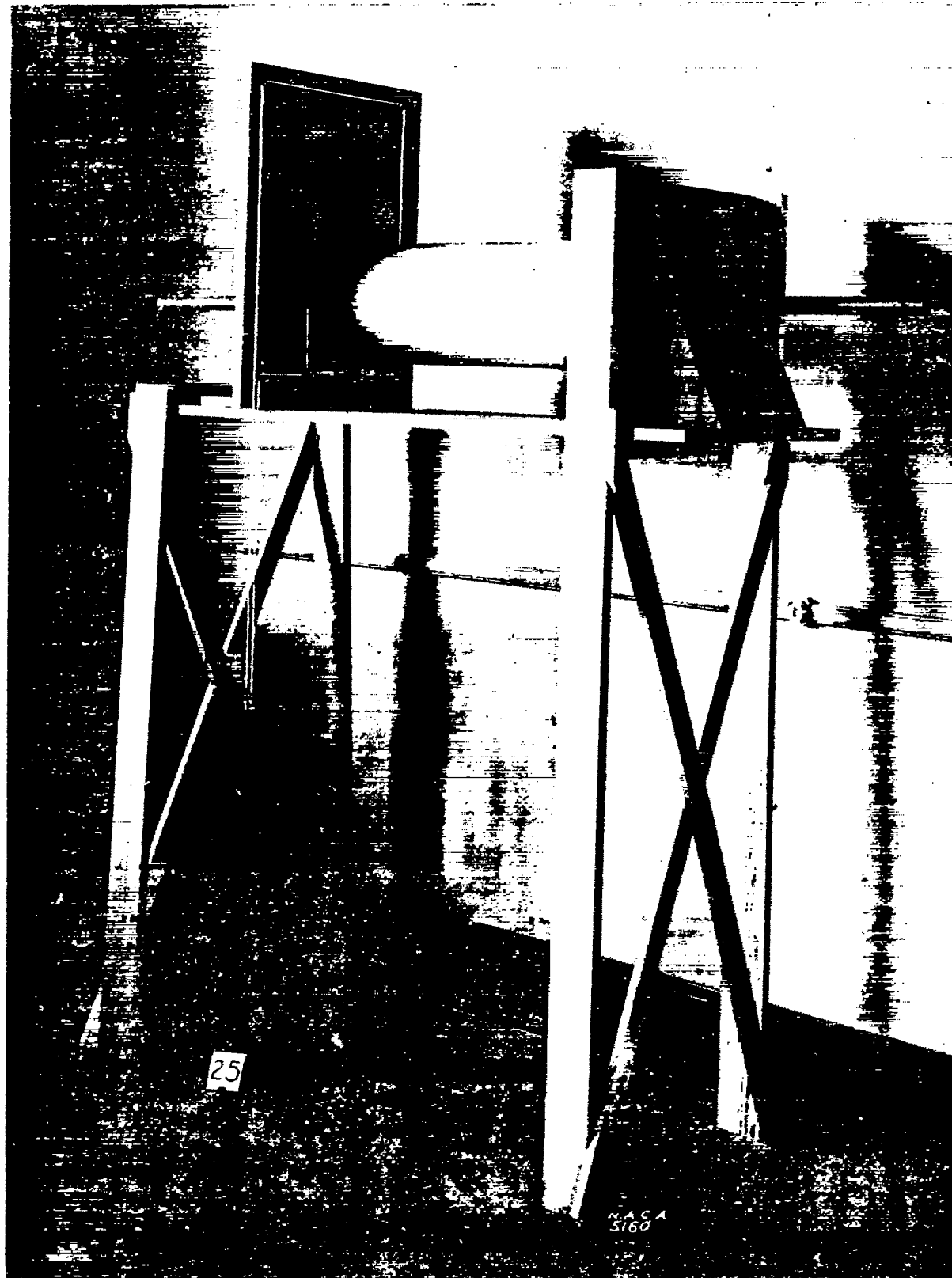


Fig.25 Testing apparatus with model in place.(Torsion).



Fig.26 Models Ta,Tb,Tc,U, and V after torsion test.Front.



Fig.27 Models Ta,Tb,Tc,U, and V after torsion test.Rear.

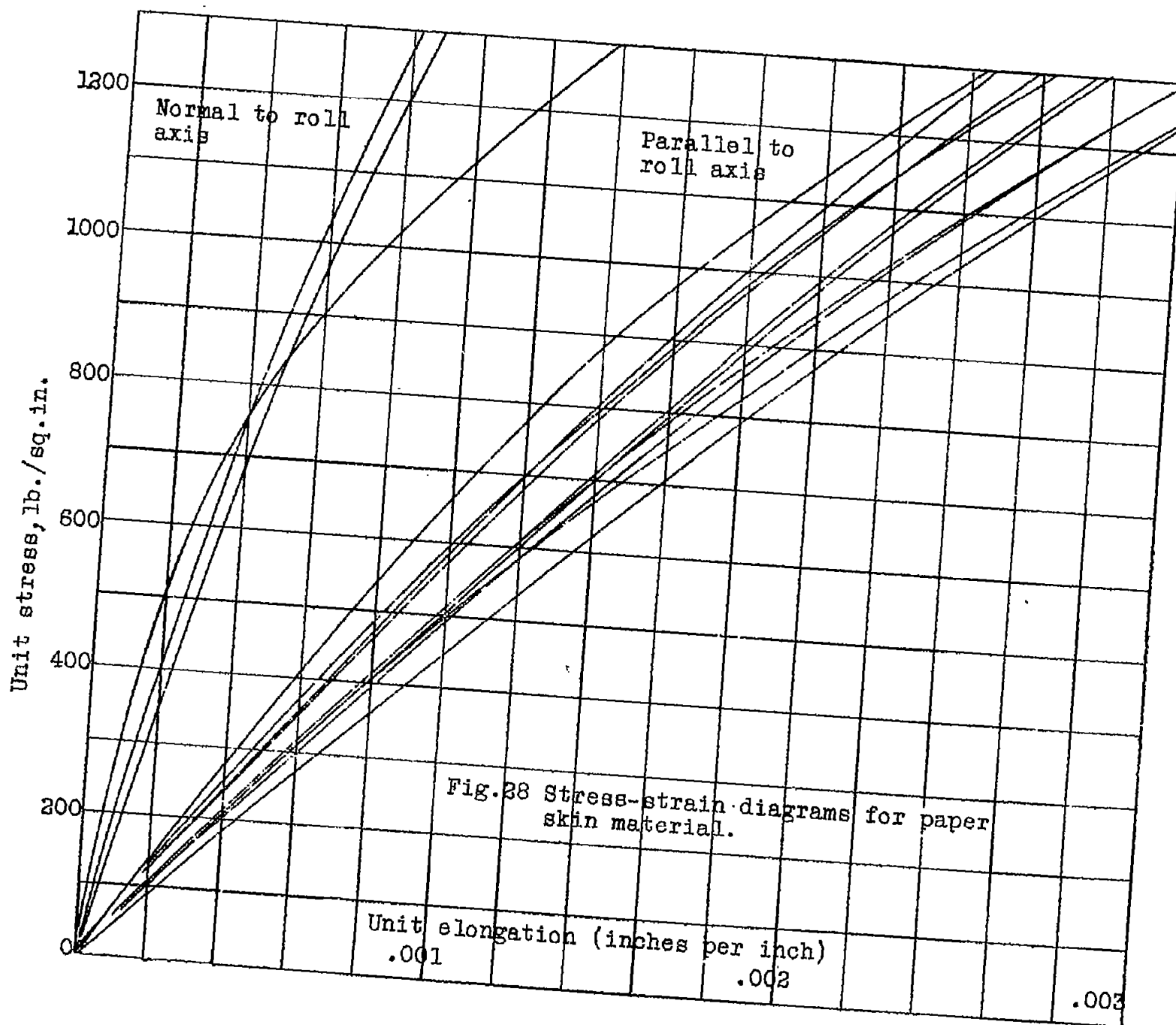


Fig. 28

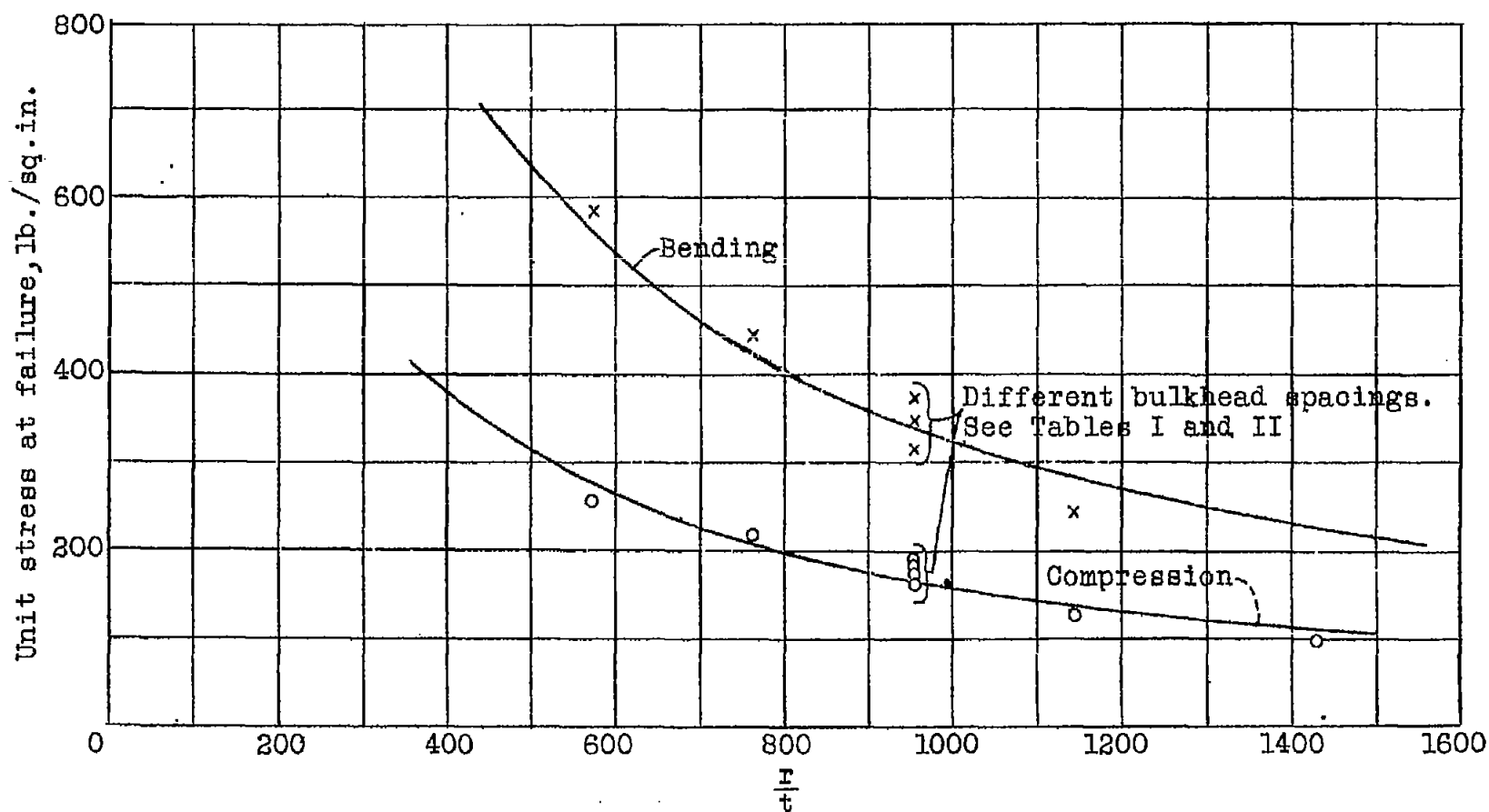


Fig. 29 Variation of unit stress with  $r/t$  in bending and compression.

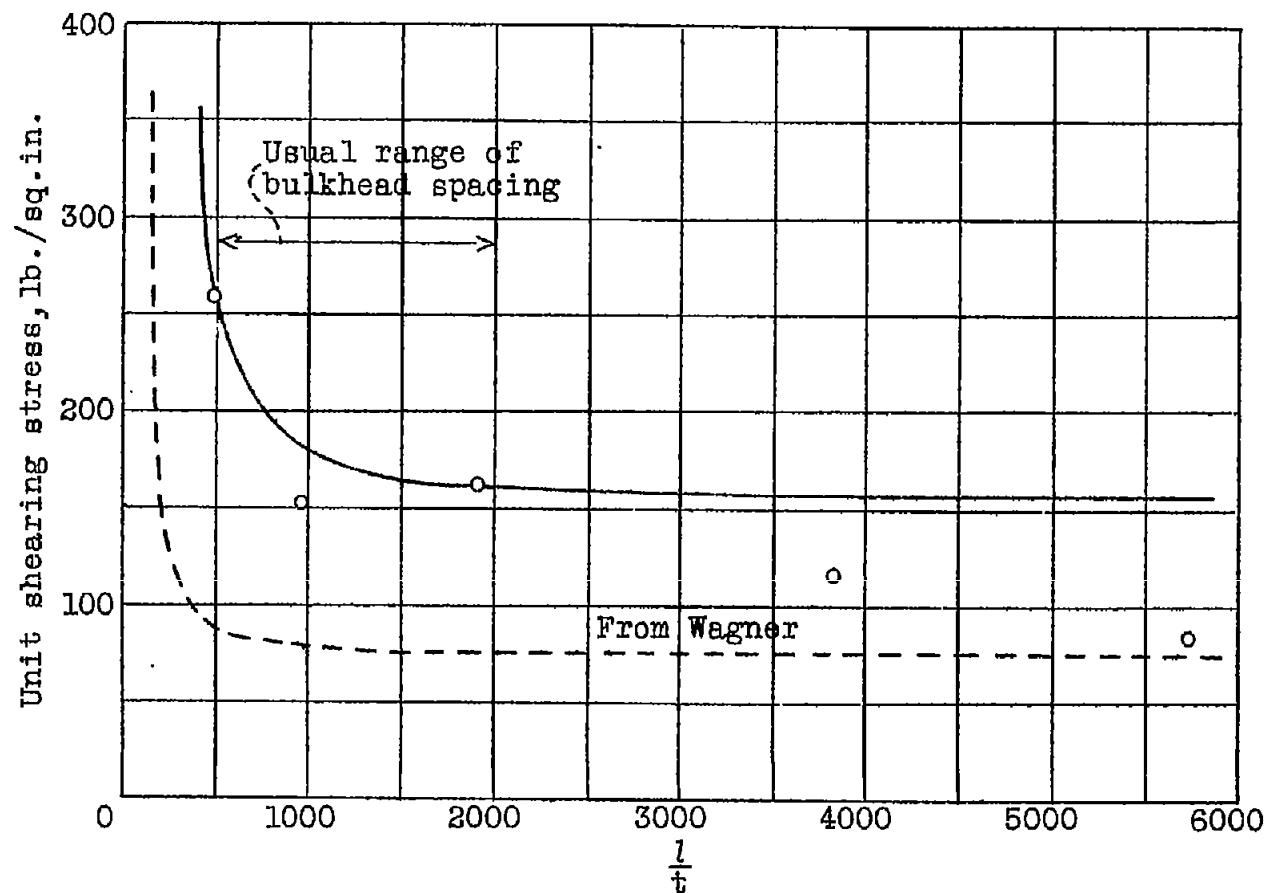


Fig.30 Effect of bulkhead spacing on strength in shear.

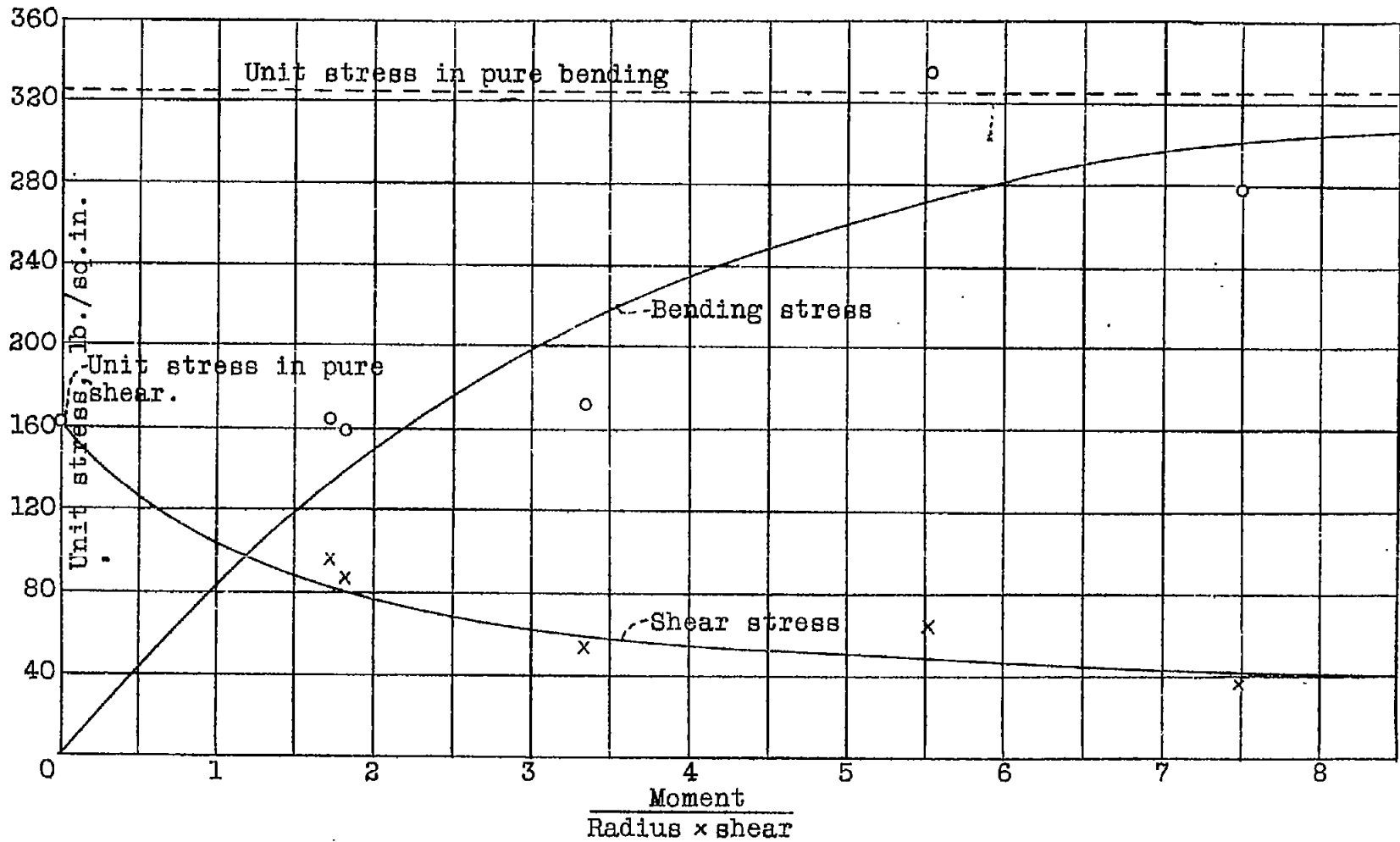


Fig.31 Stress in combined bending and shear.